THE APPLICATION OF COMMERCIAL INVESTMENT CASTING PRINCIPLES TO JEWELRY CASTING

John C. McCloskey

ABSTRACT

Investment casting as practiced in the precious metal jewelry industry has been reviewed with emphasis on (1) fluid flow, (2)heat transfer and (3) solidification events which are characteristic of jewelry casting practices. Fluid flow has been shown to be inherently turbulent by calculation of Reynolds Number in gating systems. Heat transfer conditions have been characterized and Chvorinov's law applied for considerations of freezing time in cast parts and gating systems. Solute redistribution during freezing has been calculated for solidification which is occurring under "equilibrium" and "non-equilibrium" conditions.

KEYWORDS

Precious Metals, Investment Casting, Microsegregation, Solidification, Heat Transfer.

INTRODUCTION

Investment casting of jewelry has been successfully practiced on
a large commercial scale by many companies. Until very recently,
however, most of the technical literature describing jewelry cast-
ing process technology has not included a broad treatment of basic
process principles. Commercial foundries producing "non-jewelry"
products have explicitly or implicitly supported a much more basic
treatment of casting technology. This is very evident in the
technical literature published by professional societies which
include foundrymen in their membership. Involvement with technology
has helped the producers of commercial investment castings to enjoy
annual production increases of 10-20 per cent for the last 15-20
years. Use of evaporative patterns in sand casting operations by
methods which are capable of being highly automated is experiencing
intense commercialization.

It has been unfortunate that jewelry casters have not related to
all the technical activity which has occurred in the non-precious
metal casting field. This is an intellectually distressing situa-
tion because all casting processes are governed by universal prin-
ciples. It is very gratifying to participate in this conference
which has been organized to expand our basic understanding of
casting principles as they apply to precious metals.

The kindred spirits of jewelry casters are actually found in those
investment casting foundries which are producing high quality
castings for aerospace applications. Specifications for aerospace
castings place a high emphasis on the internal soundness of a
casting. Castings are inspected for internal soundness with one
or several non-destructive testing methods such as x-rays, ultra-
sound, magnaflux, and xyglo techniques. Internal porosity, inclu-
sions, hot tears, etc. are causes for rejection of these parts.

Jewelry casters are always concerned about internal soundness in
those products which experience metal removal during cast clean-up,
polishing, chasing, and bright-cutting operations. These metal
removal operations very efficiently "inspect" castings for their
degree of internal soundness. By any objective measure, both the
aerospace part and the jewelry casting is being rejected for the
same reasons when the fundamental principles relating to internal
casting soundness are being considered. Superior quality aerospace
investment castings are produced as a result of a sound understand-
ing of all process principles involved in their production.
Application of these same principles can only be of great benefit
to the jewelry caster.

Production of superior quality castings is the result of understand-
ing the principles of (1) fluid mechanics (2) heat transfer and
(3) solidification. This paper will discuss the most important
aspects of these principles as they apply to metal casting.

FLUID FLOW

A casting requires a "plumbing" system to introduce molten metal
into a cavity which is contained in a mold that has been constructed
from a heat resistant material. This "plumbing" system is called

a "gating system" in the foundry trade. To a foundry engineer, a casting includes the part which is to be cast and the associated gating system.

Fluid flow in pipes, ducts, and gating systems associated with metal castings occurs under conditions that are described as either "turbulent" or "laminar". A judgement regarding flow conditions in any system can be made from a calculation of the Reynold's Number for the system.[1] By definition,

$$\text{Reynolds No.} = \frac{(P)\ (v)\ (d)}{u} = N_{Re}$$

where P = density of fluid
       v = fluid velocity
       d = diameter of flow channel
       u = viscosity

The flow velocity of liquid metal in gating systems can be estimated by an energy balance. The following diagram illustrates a gating system which is being filled with liquid metal. A small volume of metal at point 1 has potential energy equal to mgh relative to point 2, where m is mass, g is the gravitational constant, and h is the vertical height of point 1 above point 2.
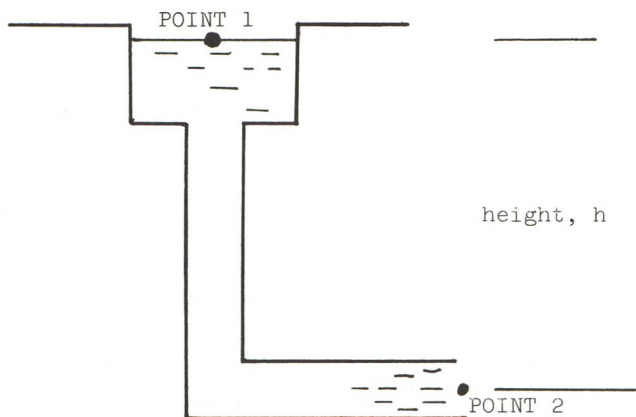

Figure 1: Flow velocity model in runner
with no frictional losses.

A small volume of metal flowing in the runner at point 2 has a kinetic energy, $\frac{1}{2}mv^2$, where m is mass, and v is velocity. Neglecting energy losses due to frictional effects, back pressure in the mold, etc., conservation of energy requires that the kinetic energy which this small volume acquires at point 2 during a descent from point 1 must equal the potential energy that it initially started

with at point 1. Or,

$$mgh = \tfrac{1}{2}mv^2$$
then
$$v = \sqrt{2gh}$$

As a numerical example, a volume of metal which experiences a vertical descent of 15 cm. during pouring acquires a velocity of 171 cm/sec, or 5 ft/sec. Resistance effects will reduce metal velocities under flow conditions experienced in actual practice.

A value for Reynold's Number can now be calculated based on this estimate of flow velocity. For a 10k yellow gold alloy

$$N_{Re} = \frac{(P)\ (v)\ (d)}{u}$$

$P = 11.3 \text{ gms/cm}^3$
$v = 171 \text{ cm/sec}$
$d = 0.50 \text{ cm } (0.20 \text{ in.})$
$u = 0.04 \text{ poise}$

$$N_{Re} = \frac{(11.3)\ (171)\ (0.5)}{0.04}$$

$$= 24,153$$

Values of Reynolds Number greater than 2,000 indicate turbulent flow conditions. Turbulent conditions prevail in gating systems under most conditions. Gating design should be such that these inherently turbulent conditions are not aggravated to the point where casting quality is caused to deteriorate. Excessive turbulence promotes surface oxidation of metal and can promote the entrapment of inclusions from mold material, slags or fluxes, and surface oxides.

The effects of turbulence can be minimized by utilization of an "unpressurized" gating system.[2] If properly used, this type of system will help make it possible to use the lowest possible metal and mold temperatures and still achieve complete mold filling.

The most general gating system which can be envisioned is comprised of a vertical sprue, horizontal runners, and horizontal gates. Schematically, it would appear as in Figure 2. Flow disruptions are promoted by a build-up of "back pressure" caused by restrictions or other abrupt changes in the geometry of a flow channel.
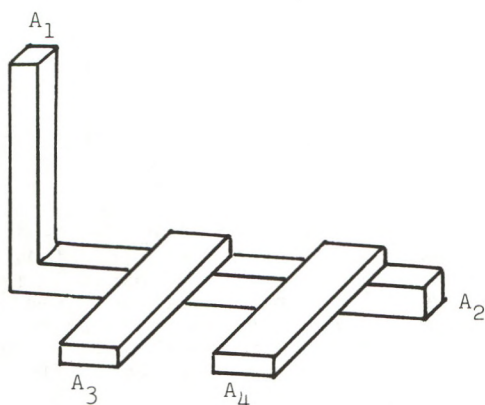
$$A_1 \leq A_2 \leq A_3 + A_4 + \cdots A_n$$

Figure 2: Schematic diagram of unpressurized gating system.

A decreasing heirarchy of flow cross sectional areas from the gates – A_3, A_4 – to runner – A_2 – and then to down-sprue A_1, will minimize metal turbulence and disruptions in the flow velocity profile caused by large changes in flow cross section.

Other design features which must be incorporated into good gating design include elimination of sharp edges and corners, and abrupt changes in flow direction.

HEAT TRANSFER

Once a mold cavity is filled with molten metal, the single major factor controlling the creation of a high quality casting is the removal of heat. Control of heat removal controls solidification and the ultimate outcome of a casting event. The change in physical state from liquid to solid during casting tends to deflect attention from the most significant change which is occurring in a solidifying system. We can directly observe the physical changes of liquid into solid. Fundamental studies of the liquid state show that the physical differences between liquid and solid are really not that great. Although thin and watery, liquid metals possess a degree of atomic order over short ranges that is similar to the high degree of long range atomic ordering found in the crystalline solids from which liquids are derived.

Understanding the significance of heat removal requires more detailed consideration.

The physical change in volume which occurs during freezing has been measured for many metals.[3] For iron, as an example, this change in volume represents a decrease in specific volume of 2.7 per cent.

Normal contraction of the solid during cooling to room temperature represents a decrease in specific volume of 10.5 per cent. The volume changes associated with actual solidification events are almost 4 times smaller than volume changes associated with simple cooling - or heating for that matter.

When we consider the energy, or heat content, of iron in the liquid and solid states, major differences are found. At the melting point of iron, we find that 1 lb. of liquid iron contains 573.75 BTU more heat than solid iron which is also at the melting point. This difference in heat content - 573.75 BTU/lb. - between liquid and solid is called the latent heat of fusion of iron. It is not the total heat content. It is only the change in heat content associated with freezing or melting. Additional heat, or energy, must be present as that amount which is required to raise the temperature of liquid iron from the melting point to a pouring temperature for casting, and the heat required to raise the temperature of the solid to the melting point.

When all heat required to solidify and cool an iron casting to room temperature is accounted for, it can be shown that 20 per cent of the heat removal requirement is represented by heat of fusion. When freezing is occurring, heat is being evolved from a casting at the melting point of the metal. Heat evolution during freezing causes the temperature of the metal to be "arrested" at and around its melting point. The implication is that during freezing and cooling of a casting, 20-30 per cent of the required heat removal must occur at or near a single temperature, the melting point of the alloy. Removing such a significant fraction of the total heat removal requirement at one temperature is a formidable task and greatly complicated by other solidification events.

Although the change in volume associated with freezing events is the major direct cause of porosity and unsoundness in castings, control of heat removal during freezing under circumstances that are inherently difficult to control is the most important method for minimizing or preventing unsoundness in cast metals.

A mold functions as a heat sink for a casting which is to be frozen. Because the thermal conductivity of metals and alloys is typically 50-100 times greater than that of a hot investment casting mold, heat removal from a typical casting could be described as "slow". The mathematics of heat transfer has been described by many authors, and it is far too cumbersome to be used as a simple working tool by the average investment caster. Because of these mathematical treatments, however, it is possible to construct a general model for heat transfer conditions in a typical jewelry investment casting process.

Heat transfer and resulting temperature distribution in mold/metal casting systems is governed by values of thermal conductivity, density, and specific heat of metal and mold. Because of the relative values of these physical properties for both metallic metals and refractory molds, the temperature of freezing metal remains fairly constant during freezing while steep temperature gradients develop on the mold side of mold-metal interfaces. The following sketch illustrates a typical temperature profile in mold and metal.
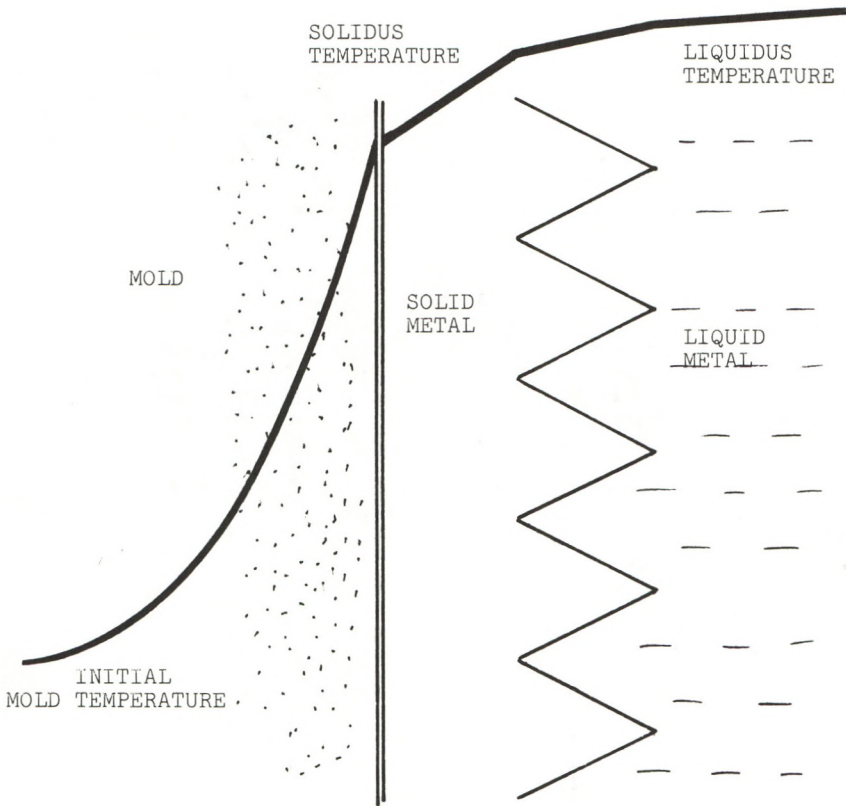
Figure 3:   Temperature profile in metal and
mold during solidification.

Ott and Raub have observed that temperature increases in the invest-
ment which surrounds cast pieces increases relatively slowly after
a casting is poured.[4] They suggest that castings of typical size
have completed the solidification process before temperature in-
creases of approximately 400°C are achieved in the mold at distances
of 1-2 mm from the casting.   Their observations are consistent with
the general temperature profile and heat flow model previously
described.

A casting becomes completely frozen when all "superheat" of the
liquid and heat of fusion has been absorbed by the mold.   The time
required to accomplish this transfer of heat is the freezing time
for the casting.

The classic work of Chvorinov first published in 1939 discloses a

useful measure of freezing time that is simple and practical.[5]
Chvorinov proposed that the freezing time of a casting is propor-
tional to the square of the ratio $\left(\dfrac{\text{volume}}{\text{surface}}\right)^2$. The logic of this
expression is to be seen by considering that the physical quantity
of heat to be removed from a casting must be directly proportional
to the mass or volume of material undergoing solidification. This
heat which must be removed from the casting must flow through the
metal, across the mold-metal interface, and into the mold. A
casting with a large surface area has a greater ability to dissipate
and transfer heat into a mold than a small casting. Hence, solidi-
fication time would be expected to be inversely proportional to
casting surface area.

Chvorinov's rule is a useful tool to apply to gating systems.
Production of castings with superior internal soundness requires
liquid metal to be made available to feed solidification shrinkage
in the casting. The gating system can be utilized to feed this
solidification shrinkage if gates do not freeze shut before the
casting completely freezes. An example of Chvorinov's law used to
"size" a gating system is as follows.

Consider a casting shaped like a thin, flat, circular disc and a
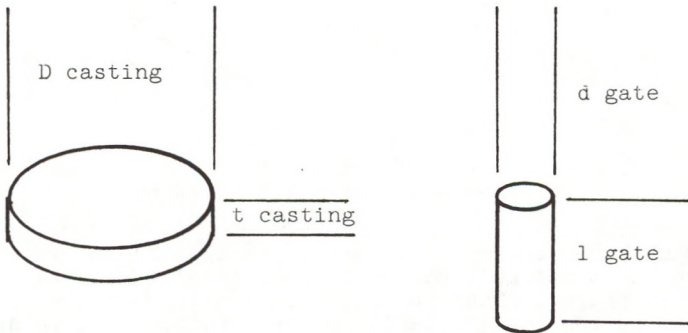cylindical gate section, each with dimensions as indicated in the
following sketch.

Figure 4: Geometrical model for disc-shaped
casting and cyclindical gate.

For the casting,

$$V_c = \frac{\pi D_c^2}{4} t$$

$$A_c = (2)\left(\frac{\pi D_c^2}{4}\right) + \pi D_c t$$

where V_c, A_c equals the volume and surface area, respectively, of the casting in accordance with simple geometric principles.

Similarly for the gate section,

$$V_G = \frac{\pi d_G^2}{4} l$$

$$A_G = \pi d_G l$$

where V_G and A_G represent the volume and surface area of the gate.

Please note that the circular area of the ends of the gate is not included in the expression for the gate surface area. These ends are going to be attached to the casting and the sprue. Heat transfer across these connection points is going to be negligible compared to the heat flow which will be occurring into the mold.

Expressions for volume and surface area can now be substituted into the expression for Chvorinov's law, $\theta_f = \left(\frac{V}{A}\right)^2$

For the casting,

$$\theta_{fc} = \left(\frac{V_c}{A_c}\right)^2 = \left[\frac{\frac{\pi D_c^2}{4} t}{\frac{\pi D_c^2}{2} + \pi D_c t_c}\right]^2$$

and in simplified form for engineering estimates,

$$\theta_{fc} = \left(\frac{V_c}{A_c}\right)^2 = \left(\frac{t_c}{2}\right)^2$$

It must be noted in the above equation that the term $\pi D_c t_c$ has been eliminated. For a casting in which the diameter, D_c, is much greater than the thickness, t_c, the value of $\pi D_c t_c$ will be negligibly small compared to other terms in the expression.

For the gate section,

$$\theta_{fG} = \left(\frac{V_G}{A_G}\right)^2 = \left[\frac{\frac{\pi d_G^2 \, 1}{4}}{\pi d_G \, 1}\right]^2$$

which simplifies to

$$\theta_{fG} = \left(\frac{d_G}{4}\right)^2$$

Once again, if the gate connecting the casting to a sprue post is to be effective in allowing liquid metal in the sprue to feed solidification shrinkage in the casting, the freezing time of the gate must be equal to or greater than the freezing time of the casting, or θ_f(gate) $\geq \theta_f$(casting). If we substitute the simplified relationships which were previously developed for freezing time, then

$$\theta_f(\text{gate}) \geq \theta_f(\text{casting})$$

$$\left(\frac{d_G}{4}\right)^2 \geq \left(\frac{t_c}{2}\right)^2$$

$$\frac{d_G}{4} \geq \frac{t_c}{2}$$

$$d \text{ gate} \geq 2t_c$$

This design exercise suggests that a gate must have a diameter which is twice the thickness of the cast part. Basically, control of casting soundness is achieved by control of heat removal from the casting. An understanding of heat transfer principles is very helpful in maximizing the degree of control that can be ultimately achieved.

SOLIDIFICATION OF METALS

Previous discussions describing the flow of heat out of castings describes the basic method by which castings are caused to freeze.

In general, the solidified skin of a casting will advance into the hotter, interior liquid mass of metal such that the thickness of the solid layer is proportional to the square root of time, or:

$$S = k\sqrt{t}$$

> where s is thickness of solid
> t is time elapsed since on-set of freezing
> k is very close to 1 in many mold/metal systems.

This picture of a distinct solid interface advancing into a liquid is idealized and can only be made to happen on a macro scale by imposing an extremely steep temperature gradient across a freezing

solid-liquid interface. The physical mechanisms by which alloys freeze promotes instability and breakdown of flat, smooth solid/ liquid interfaces if they try to form. Phenomena which surround the events of solidification will be discussed as the final phase of our efforts to understand the production of superior quality castings.

Solidification is a nucleation and growth process. The first metal to form in a freezing process is called a nucleus. It can form spontaneously or be created with some type of articicial stimulation - a cold mold wall, an inoculant of grain refiner, etc. Growth of nuclei create distinct grains of metal. It must always be remember- ed that this growth process is actually occurring on a surface. Atoms are leaving the liquid state and taking up residence on a solid surface by a process that looks like vapor deposition. The rate at which atoms can leave the liquid state is governed by the rate at which they lose their heat of fusion which in turn depends on the rate of heat removal from the freezing casting.

Solidifying grains of metal assume a dendrite structure. A dendrite looks very much like a pine tree and the word dendrite is derived from the Greek word "dendros", meaning tree.

One reason dendrites form is because the solid metal can maximize the surface area which is available for deposition of atoms from the liquid state. Dendrites represent a collection of mutually perpendicular arms in cubic metals and the spacing between adjacent arms is a measure of local cooling rates during freezing. Fine dendrite arm spacings represent rapid cooling, while large arm spacings represent slow cooling. Arm spacings generally fall within the range of 10 microns to 300 microns.

Alloys tend to "unmix" themselves when they freeze. This fact can be seen on any phase diagram for a binary alloy. Figure 5 is the phase diagram for gold and copper.
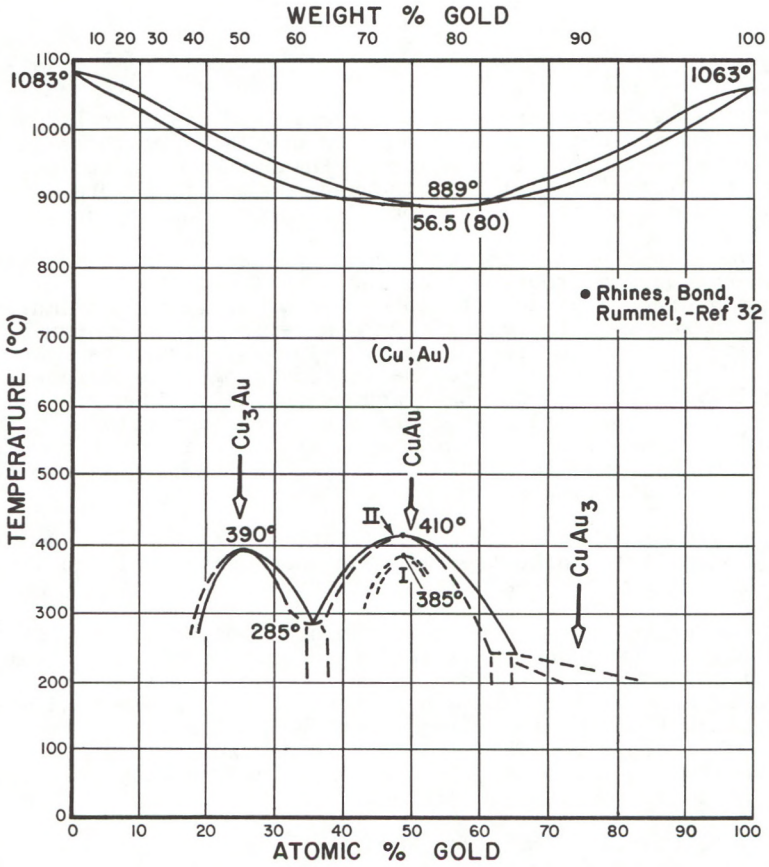
Figure 5: Gold-Copper Phase Diagram

From M. Hansen, "Constitution of Binary Alloys", McGraw Hill, New York, 1958.

A phase diagram describes the characteristics of various phases in metal alloys which coexist at various temperatures. Our discussions will be confined to relationships which exist between solid and liquid phases during solidification of the liquid.

In Figure 5, freezing starts at the liquidus for any alloy and a specific liquidus temperature exists for specific alloys. For example, an alloy of 90% Gold - 10% Copper exibits a liquidus temperature of 945°C.

At the liquidus temperature, the over-all, average composition of the liquid in an alloy mass which is just starting to freeze is the same as the initial composition of the alloy. The composition of the solid which starts to form is much different from the liquid, however. The composition of the solid, as demonstrated on a phase diagram, is given by the intersection of a horizontal temperature line with the "solidus" line on the phase diagram when this temperature "horizontal" is drawn at the liquidus temperature.

Figure 6 is a graphical demonstration of the construction of a temperature horizontal to determine the composition of solid and liquid which co-exist at the liquidus temperature in an Aluminum - 4.5% copper alloy.[6]

Such a horizontal temperature line is also called a "tie line". In two component alloys, composition at the ends of tie-lines can be determined directly from the phase diagram of any alloy system. When more than two components are present in an alloy, tie lines must be experimentally determined.

As solidification proceeds and the temperature of a freezing alloy mass declines, the composition of the liquid traverses the liquidus line and the composition of the solid traverses the solidus line. These continuous changes in the chemical composition of solid and liquid promote what is called "coring" in castings. When combined with convection currents in the liquid, gross chemical segregation effects start to occur. Also, localized gradients in chemical composition at the surface of a freezing solid help promote and stabilize dendritic growth structures.

In spite of appearing very dynamic and complex, some mathematical relationships can be derived that explain a great deal about the events occurring during freezing.[6] These relationships can help explain some of the difficulties which can be encountered during the production of high quality castings.

Based on the recognition of the fact that liquid and solid composition is changing continuously during freezing, two simple, yet "extreme" models can be constructed to help describe solidification events.

One simple model for freezing assumes that atoms have infinite mobility in both the liquid and solid state. Separately, within the solid and liquid, compositional differences are eliminated as rapidly as they are created. With such a model, the overall composition of the solid traverses the solidus line on a phase diagram and the overall composition of the liquid traverses the liquidus
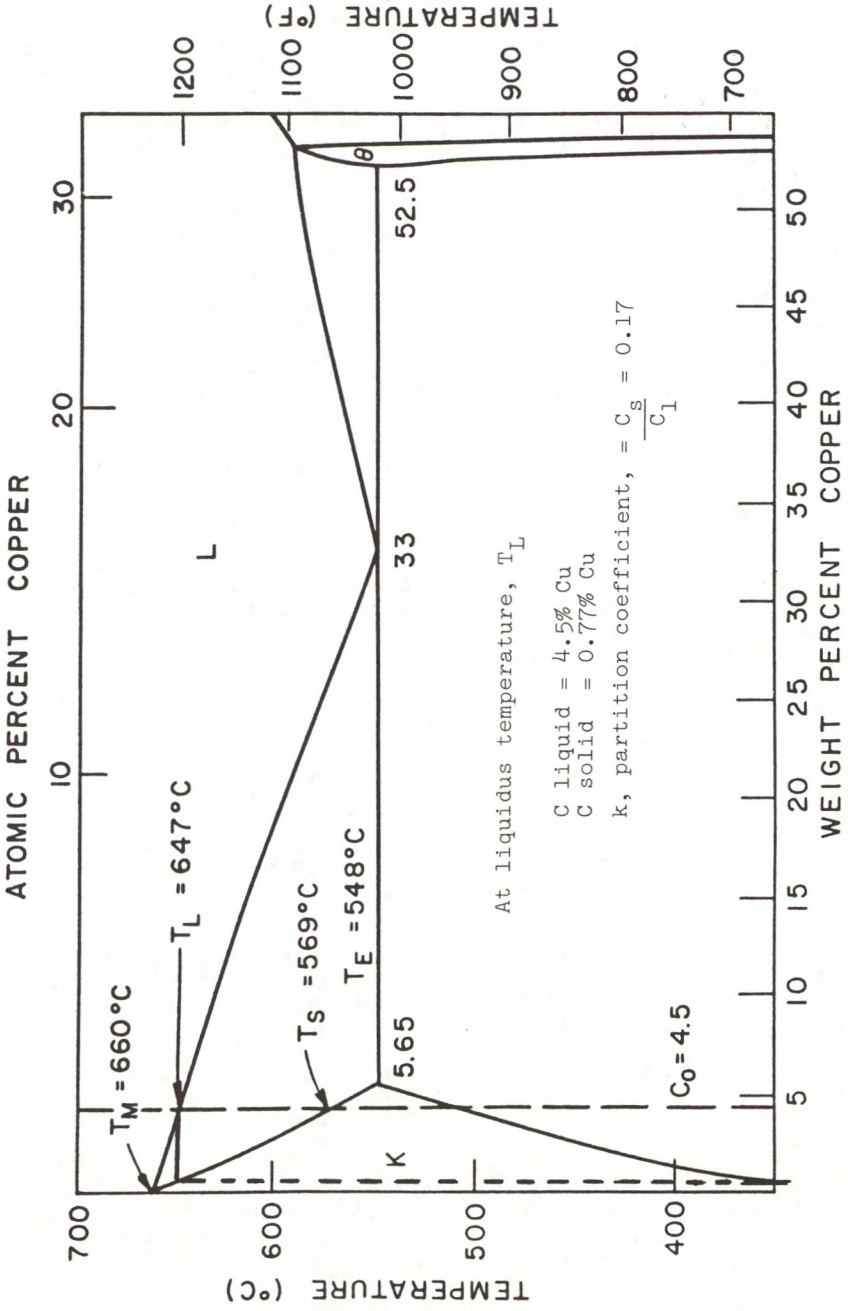
Figure 6: Tie line construction at liquidus temperature of an Aluminum – 4.5% Copper Alloy.

line.   The composition of the solid is different from the composi-
tion of the liquid, but no variations in composition exist within
the solid or the liquid.   Basically, this model describes solidifi-
cation under "equilibrium" conditions.

A second model for freezing can be constructed by recognizing that
atoms do not move very rapidly in the solid state.   A "non-equilib-
rium" model for solidification then becomes one which assumes no
atomic diffusion in the solid with infinite diffusion in the liquid.
Composition variations develop and are allowed to exist within the
solid phase with this model.

A simple material balance can be used to derive mathematical expres-
sions for fraction of solid in a freezing system as a function of
temperature and composition conditions in the solid and liquid
during freezing.   These expressions apply to the casting as a whole
because they are constructed on the basis of the dendritic structure
of solidifying metal.   Since the primary growth form of the solid is
used as the basis for these mathematical models, the models then
apply by extension to a casting of any "real" physical size.

The detailed derivations of these mathematical expressions can be
found in the literature.   The development applies only to binary
alloys.

For solidification occurring under conditions that were described
for equilibrium solidification, fraction of solid as a function of
temperature is described by the relationship

$$f_s = \frac{T_L - T}{(T_m - T)(1 - k)}$$

and for non-equilibrium solidification,

$$f_s = 1 - \left(\frac{T_m - T_L}{T_m - T} \right)^{\frac{1}{1 - k}}$$

where f_s = fraction of solid
        T_m = melting point of solvent element
        T_L = liquidus temperature of alloy
        T = temperature at which f_s is to be determined
        k = ratio $\frac{C_s}{C_L}$, (C_s = composition of solid in
              equilibrium with liquid composition, C_L),
              k is assumed to be a constant

These expressions have been extensively evaluated by experimental
observation in the Aluminum - 4.5 per cent copper alloy system.
Figure 7 is a plot of fraction solid versus temperature for Al - 4.5
per cent copper alloys with a presentation of experimental observa-
tions.[6]   The mathematical model for non-equilibrium freezing predicts
the formation of Cu - CuAl$_2$ eutectic in this alloy during freezing.
Equilibrium considerations predict that no eutectic should be
present.   Actual experimental observations revealed the presence of
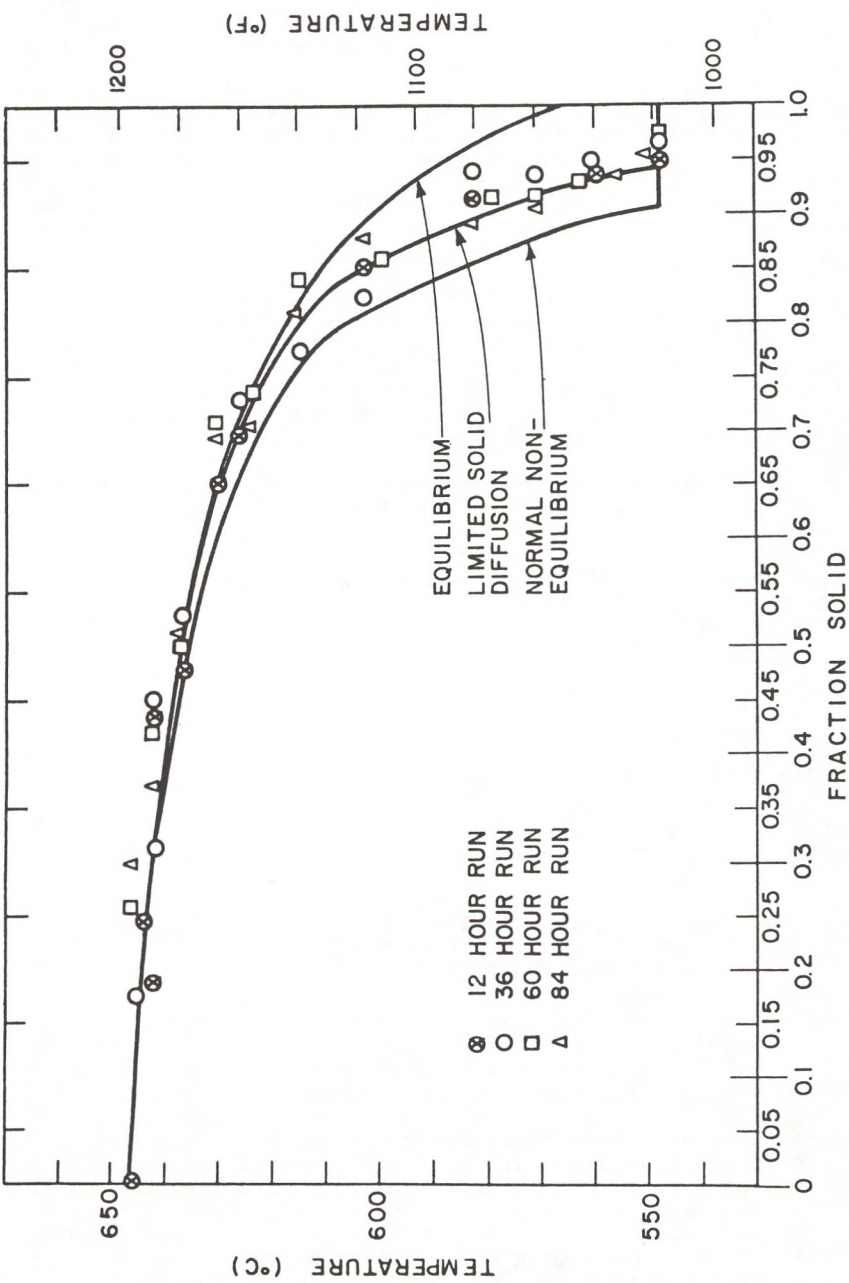eutectic.   Events during freezing were more closely predicted by

Figure 7: Fraction solid versus temperature in Aluminum – 4.5% Copper alloy for equilibrium and non-equilibrium freezing conditions with data from controlled solidification experiments.

the considerations of non-equilibrium freezing events than by an
equilibrium model.

An attempt has been made to apply mathematical equations describing
the effects of solute redistribution during freezing to jewelry
alloys. Several assumptions have been made to simplify calculations,
and to compensate for lack of information.

Since the mathematics of solute redistribution has been derived for
binary alloys, an 18 kt gold alloy containing 75% gold, 16% silver,
9% copper is assumed to be an alloy of 91% gold - 9% copper.
Partition ratio, k, is assumed to be constant, although the curved
liquidus and solidus boundaries of the Au - Cu phase diagram indi-
cates that this is not true. Straightening the phase boundaries to
develop a constant value for k resulted in the use of a fictitious
value of 1050°C for the melting point of gold.

Figure 8 is a plot of fraction solid versus temperature for Au - 9%
Cu alloys under equilibrium and non-equilibrium freezing conditions.
The non-equilibrium freezing model predicts solute build-up to the
monotectic composition 80% Au- 20% Cu. Equilibrium freezing con-
siderations do not predict this build-up, of course.

Expressions for composition of solid at the solid-liquid interface
as a function of fraction solid for equilibrium and non-equilibrium
freezing are as follows:

Equilibrium freezing:

$$C_S = \frac{k\,C_O}{1 - f_S\,(1 - k)}$$

Non-equilibrium freezing:

$$C_S = k\,C_O\,(1 - f_S)^{k - 1}$$

where C_S = composition of solid
        C_O = initial composition of alloy
        f_S = fraction of solid
        k = partition ratio, $\dfrac{C_S}{C_L}$, a constant

Data used to generate fraction solid versus temperature plots was
used to calculate composition of solid as a function of fraction
solid. Results are presented in Figure 9. The model for non-
equilibrium freezing predicts a build-up of copper to the 20% copper
monotectic composition during freezing. Unfortunately, I do not
know of any experimental attempts to confirm the presence of local-
ized regions of high copper content in 18 kt gold alloys which
contain copper.

SUMMARY

It has been demonstrated that many casting practices which are
routinely utilized to cast precious metals into decorative jewelry
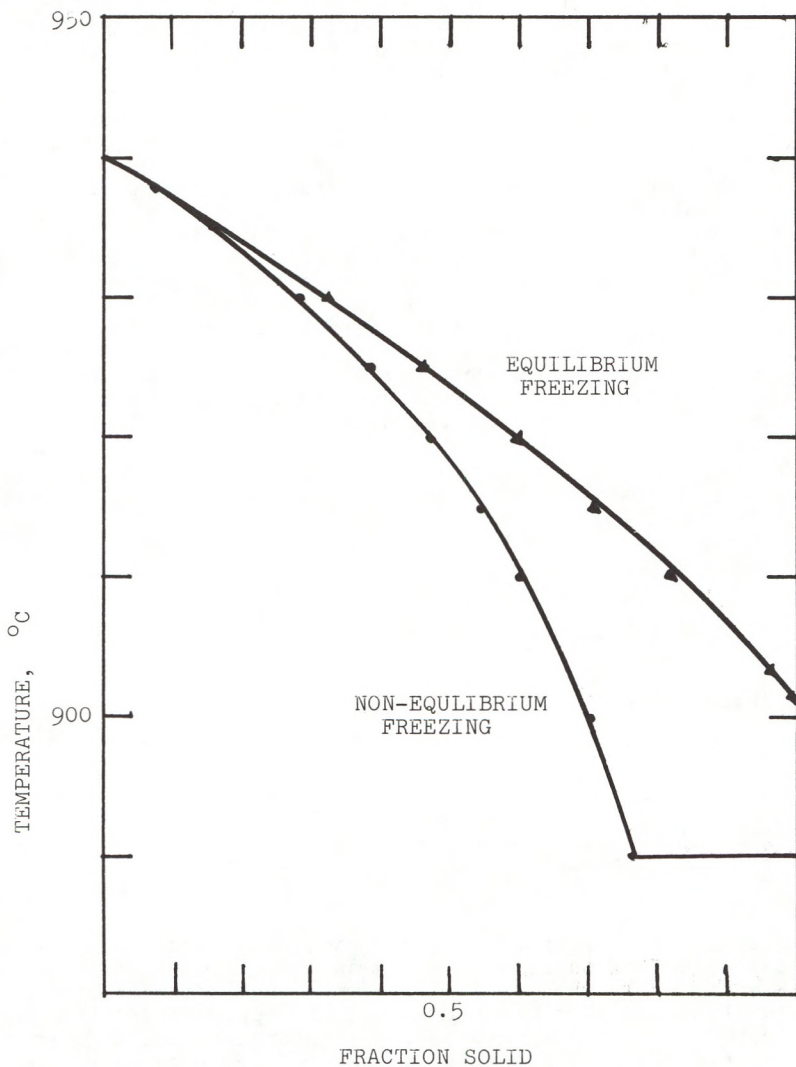products have a fundamental basis in sound engineering principles.

Figure 8: Fraction solid versus temperature in Gold - 9% Copper alloys under equilibrium and non-equilibrium freezing conditions.
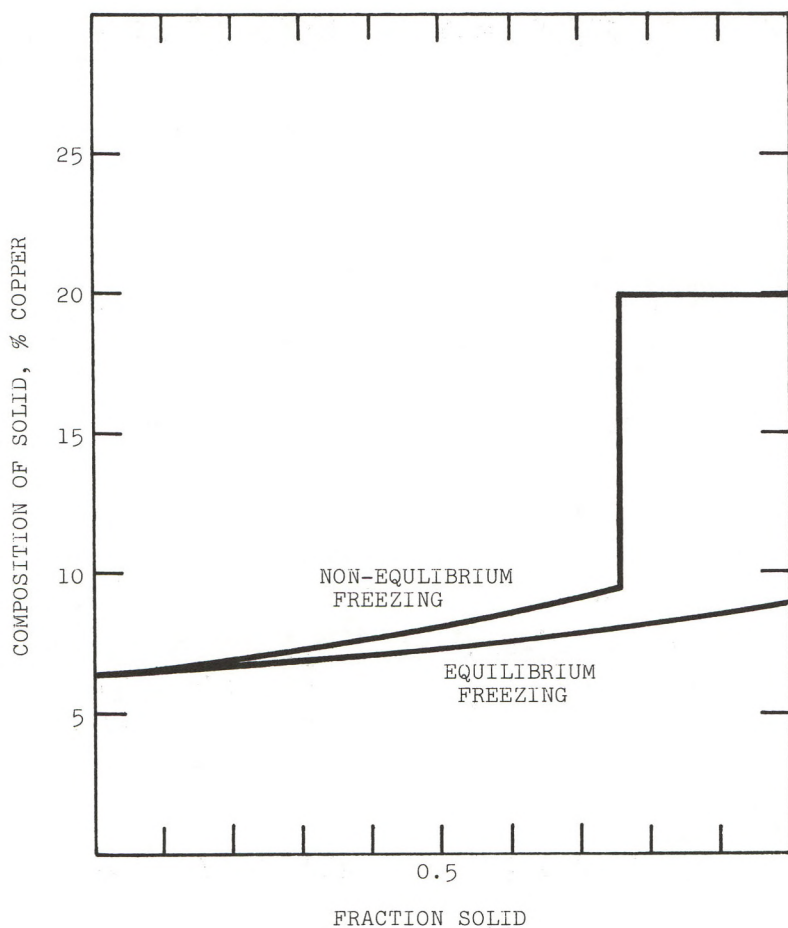
Figure 9: Composition of solid versus fraction
solid in Gold – 9% Copper alloys
freezing under equilibrium and
non-equilibrium conditions.

The specific engineering principles that are most pertinent for jewelry casting include (1) fluid mechanics, (2) heat transfer and (3) liquid to solid transformations, or, solidification. Continued efforts by jewelry casters to see beyond the routine of traditional, "customary" practices will develop better insights into practical solutions to the causes for product failure which arise in casting processes. An additional outcome of such efforts must inevitably be a rise in levels of productivity, the development of new and better alloys and materials for use in casting processes, and the opening of a new world to the artisan/engineer who has always been the ultimate driving force in the world of jewelry casting.

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